

A Note on Spontaneous Baryogenesis
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Abstract

The original calculation of ‘spontaneous’ baryogenesis overlooked the role played by transport of particles onto the bubbles wall. For typical ‘adiabatic’ wall thicknesses and velocities one can model the problem in a fluid approximation and the mechanism is best understood as another limit of ‘charge transport’ baryogenesis, and can produce a similar non-local effect in front of the wall.

In this brief talk I will make a few comments on ‘spontaneous baryogenesis’, a mechanism for the production of baryons at the electroweak phase transition in the ‘adiabatic’ limit of thick slowly moving bubble walls proposed by Cohen, Kaplan and Nelson (CKN) in [1]. My comments summarize results of a collaboration with Tomislav Prokopec and Neil Turok. The central point is that the constraints imposed on particle numbers in the thermodynamic equilibrium calculations which this mechanism involves are inappropriate for the relevant wall thicknesses. A correct calculation of the baryon production involves taking into account the effect on the surrounding plasma of what is effectively a background axial gauge field on the wall. This calculation should really be thought of as another limit of ‘non-local’ baryogenesis, advocated by CKN in the case of very thin bubble walls and discussed by my collaborators in other talks at this conference [2], rather than as a distinct mechanism.

Consider the familiar two Higgs doublet model in which the CP violating phase θ changes on the bubble wall during the phase transition. If, for simplicity, we couple only one of the two doublets to the fermions through Yukawa terms, the mass terms which result when the Higgs doublet acquires a vev contain the (gauge invariant) phase θ and may be written (in the appropriate gauge)

$$Yve^{i\theta}\bar{q}_Lq_R + Y've^{-i\theta}\bar{q}'_Lq'_R + h.c. \quad (1)$$

where q stands for the charge $\frac{2}{3}$ quarks, q' for the charge $-\frac{1}{3}$ quarks and the charged leptons and Y and v for the Yukawa coupling and vev.

The observation made by CKN is that if one does a hypercharge rotation (chosen because it is anomaly free) on the fermions to remove the phase θ and make the mass terms real, the kinetic terms generate

$$2\partial_\mu\theta\Sigma_i y_i\bar{\psi}_i\gamma^\mu\psi_i \quad (2)$$

where the sum is over all the fermions which have hypercharge y_i . If one now considers the case in which the spatial gradients $\partial_i\theta$ are negligible, (2) looks like a chemical potential term for fermionic hypercharge. Because it is not strictly a chemical potential - it does not arise from a constraint- CKN refer to it as a ‘charge potential’. However if one does thermodynamic calculations with this Hamiltonian the effect of the term is just like that of a chemical potential e.g. the number density n_i of a massless fermion i in the presence of a chemical potential μ_i is

$$n_i = \frac{T^2}{6}(\mu_i + 2y_i\dot{\theta}). \quad (3)$$

So how does this bias electroweak anomalous processes? As the bubble wall passes through a region this charge potential is turned on and the system will approach the local thermal equilibrium attained in this background. CKN calculate this local equilibrium in [1] by imposing constraints on the quantum numbers in the system which are conserved by the perturbative processes which are slow relative to the timescale for the wall to pass by. They do this by introducing chemical potentials μ_A to constrain each of these charges to zero (their global value). Amongst these charges is the baryon number B (because the sphaleron processes are too slow to reach equilibrium as the wall passes) and a corresponding chemical potential μ_B . For example if we take the conserved charges, for simplicity, to be Y , the total hypercharge, B and $B - L$, where L is the lepton number, we find

$$Y \propto 20\dot{\theta} + (10 + n)\mu_Y + 8\mu_{B-L} + 2\mu_B = 0$$

$$\begin{aligned} B - L &\propto 16\dot{\theta} + 8\mu_Y + 13\mu_{B-L} + 4\mu_B = 0 \\ B &\propto 2\dot{\theta} + \mu_Y + 2\mu_{B-L} + 2\mu_B = 0. \end{aligned} \quad (4)$$

using the relation $\mu_i = \Sigma_A q_i^A \mu_A$ and $n_i \propto k_i \mu_i$ for the particles which we assume massless. k_i is a counting factor which is one for fermions and two for bosons, q_i^A is the Q_A charge of species i , and n is the number of Higgs doublets in the theory. Solving, we find $\mu_B \propto \dot{\theta} n$.

To find the resulting B violation we use the fact that the rate of change of a ‘conserved’ charge X due to a ‘slow’ process is

$$\dot{X} = -\Gamma \frac{\delta F}{T} \delta X \quad (5)$$

where δF and δX are the changes in the free energy and in X per process respectively and Γ is its equilibrium rate. Applying this to the electroweak sphaleron processes we have

$$\dot{B} = -\Gamma_{sph} N_F^2 \frac{\mu_B}{T} \quad (6)$$

where Γ_{sph} is the equilibrium electroweak sphaleron rate and N_F the number of families. We conclude that B violation on the wall is directly driven by $\dot{\theta}$ and the final generated asymmetry is simply calculable from (6).

There is a simple explanation of the proportionality of the answer to the number of Higgs doublets. In the local thermal equilibrium in which each particle species S_i has chemical potential μ_i the rate at which any particle density n_i changes due to some out of equilibrium process $\nu_j S_j \rightarrow 0$ (where, for example, $A \rightarrow 2B + C$ has $\nu_A = 1, \nu_B = -2, \nu_C = -1$) is, by (5) above,

$$\dot{n}_i = -\frac{\Gamma}{T} (\Sigma_j \nu_j \mu_j) \nu_i. \quad (7)$$

It follows from this that the particle densities are only affected directly when $\dot{\theta}$ is turned on by processes which have Higgs particles in the ingoing or outgoing state. Any other process obeys $\Sigma_i \mu_i \nu_i = 0$ simply by conservation of fermionic hypercharge, and therefore by (7) is not out of equilibrium when $\dot{\theta}$ is turned on. In CKN’s calculation it is the top/Higgs coupling which is taken to be ‘fast’. Forcing this process to be in equilibrium alters the abundances in such a way as to produce a non-zero rate for the out of equilibrium sphaleron processes.

It is now instructive to compare this with the case of a potential $\phi_Y(x)$ for *total* hypercharge which is turned on in some region of space. When such a potential is turned on no process is out of equilibrium and hence no density directly affected, simply because total hypercharge is conserved in all processes. However this does not mean that the densities of particles do not change if the potential is turned on in some small region. In fact the potential will be screened by drawing in charge from the plasma. The local equilibrium, which is a solution of the Boltzmann equation, is given by the particle phase space distributions f_i

$$f_i(p, x) = \frac{1}{e^{\beta E_i(x)} \pm 1} \quad (8)$$

where $E_i = \sqrt{p^2 + m_i^2} + y_i \phi_Y(x)$ and, therefore, $n_i \propto y_i \phi_Y(x)$.

In the case of the charge potential $\dot{\theta}$ there is also precisely such an unconstrained equilibrium solution, except that E_i is now the energy level of species S_i in the

presence of $\dot{\theta}$. If this equilibrium is attained neither the Higgs processes nor the sphaleron processes are out of equilibrium. In the same way as in the presence of a total hypercharge potential there can be transport processes which bring the system towards the static equilibrium (8).

So the central question is whether there can be significant transport of charge into the small region (the bubble wall during the first order phase transition) over the relevant timescale. A simple estimate is as follows. The distance a particle with diffusion constant D typically diffuses in a given direction in time t is $\sqrt{2Dt}$. In order that the transport processes be negligible the distance a particle diffuses in the time the wall of length L moving with velocity v_w takes to pass should be much less than the wall thickness i.e.

$$L \gg \frac{2D}{v_w}. \quad (9)$$

For quarks $D \sim \frac{10}{T}$ and in order to allow the Higgs/top processes, for which $\Gamma \sim \frac{T}{50}$, time to equilibrate we need a small v_w for a typical ‘thick’ wall with $L \sim \frac{20-40}{T}$. For typical ‘adiabatic’ wall thicknesses and velocities therefore the constrained equilibrium calculated by CKN is not attained.

A more quantitative analysis requires a full treatment which we will present elsewhere [3], but bears out this rough qualitative argument. The problem becomes that of the determination of the deviations caused by the motion of the wall from the equilibrium (8) when $\frac{v_w L}{D} < 1$. Given that the wall is thicker than the mean free path of the quarks it is valid to treat the problem in a fluid approximation. The effect of the charge potential $\dot{\theta}$ - or more generally of $\partial_\mu \theta$ - is to both alter the rates of processes coupling the fluids locally *and* to induce a force pulling or pushing the fluids around on the wall.

To conclude we return to the lagrangian after the hypercharge rotation which can be written

$$\bar{\psi}(i\partial + \frac{y_L + y_R}{2}\partial\theta - \frac{y_L - y_R}{2}\gamma^5\partial\theta)\psi + m\bar{\psi}\psi. \quad (10)$$

The vector part of the induced term can be rotated away by making use of the surviving electromagnetic vector $U(1)$ ¹. One can see explicitly that the effect of the changing phase is to turn on an axial $U(1)$ gauge field, which despite the fact that it is pure gauge cannot be removed because the axial symmetry is explicitly broken by the mass term.

The physical effects of this field can be understood in various different limits. In fact the calculation of the solutions for a Dirac particle in this background is what is done to find the reflection coefficients for the different chirality quarks or leptons in the ‘charge transport’ mechanism. In the present case where the wall is thick and the particles cannot be treated as free we need to understand the dynamics of the particles while they are on the wall. Working in the WKB limit (which should be good for most of the particles on the wall if we take $\partial_z \theta \sim L^{-1} \ll T$) one can calculate an effective chiral ‘classical’ force, which one then uses in the fluid equations. This can lead to a chiral disturbance *in front of* the bubble wall, just as in the ‘charge transport’ mechanism. In contrast to that case the chiral asymmetry in front of the wall is not driven by a reflected flux of particles with $p_z \sim L^{-1}$ but by the effect on the fluid of a build-up of particle densities on the wall due to a classical force which acts on particles of *all momenta* as they cross the wall. Despite the fact that the reflection is very suppressed for thick walls the same $\partial_\mu \theta$ field can in this case produce a chiral

¹The corresponding ambiguity about the original rotation was ignored in our discussion above.

asymmetry and ‘non-local’ baryogenesis in front of the wall through its effect on a different part of the phase space. We will explore this limit of ‘classical non-local’ baryogenesis in a forthcoming publication.

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References

- [1] A. Cohen, D. Kaplan and A. Nelson, Phys. Lett. **B263**, 86 (1991).
- [2] See the talks by N. Turok and T. Prokopec in this volume.
- [3] A sketch of the fluid calculation is given in M. Joyce, T. Prokopec and N. Turok, Princeton Preprint PUP-TH-1436(1993) and will be presented in detail in a forthcoming publication.